Mark Scheme (Results)

Summer 2017

Pearson Edexcel International A Level
In Core Mathematics C34 (WMA02/01)

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Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 125.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes..

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper or ag- answer given
- $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c|, \quad \text{leading to } x = \ldots$$

$$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where } |pq|=|c| \text{ and } |mn|=|a|, \quad \text{leading to } x = \ldots$$

2. Formula

Attempt to use correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\quad (x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0, \quad$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question Number	Scheme	Marks
1	$3x^2+2xy-2y^2+4=0 \Rightarrow 6x+2x\dfrac{dy}{dx}+2y-4y\dfrac{dy}{dx}=0$	B1 M1 A1
	Sets $x=2, y=4 \Rightarrow 12+4\dfrac{dy}{dx}+8-16\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=\dfrac{5}{3}$	M1
	Uses $x=2, y=4$ and their $\dfrac{dy}{dx}=\dfrac{5}{3} \Rightarrow (y-4)=\dfrac{5}{3}(x-2)$	M1
	$5x-3y+2=0$	A1
		(6 marks)

B1: $2xy$ differentiated correctly to give $2x\dfrac{dy}{dx}+2y$ or any equivalent correct expression.

M1: Attempts to apply the chain rule to $-2y^2$ to give an expression of the form $Ay\dfrac{dy}{dx}$

A1: Fully correct differentiation of $3x^2-2y^2+4$ to give $6x-4y\dfrac{dy}{dx}$ and "= 0" which may be implied by subsequent work. "= 0" may also be implied if the candidate rearranges the given equation first.

Allow the candidate to start with $\dfrac{dy}{dx}=...$ for all the above marks but if **this** $\dfrac{dy}{dx}$ is used to find the gradient, the next mark would be withheld as the two $\dfrac{dy}{dx}$ terms must come from the $2xy$ and $2y^2$ terms – see below.

Note: If $6x\,dx+2x\,dy+2y\,dx-4y\,dy=0 \Rightarrow 6x+2x\dfrac{dy}{dx}+2y-4y\dfrac{dy}{dx}=0$ is seen, score B1 for $2x\,dy+2y\,dx$ and M1 for $6x\,dx-4y\,dy=0$ then A1 for $6x+2x\dfrac{dy}{dx}+2y-4y\dfrac{dy}{dx}=0$

M1: Substitutes $x=2$ and $y=4$ and attempts to find $\dfrac{dy}{dx}$ (this may be implied e.g. they may rearrange their $\dfrac{dy}{dx}$ to find $-\dfrac{dx}{dy}$ and then substitute). This is not formally dependent on the first M but is dependent upon them having two $\dfrac{dy}{dx}$ terms in their derivative. One coming from $2xy$ and one coming from $2y^2$.

M1: Uses $x=2$ and $y=4$ and their numerical value of $\dfrac{dy}{dx}\left(=\dfrac{20}{12}=\dfrac{5}{3}\right)$ to find an equation of a tangent (not a normal). If $y = mx + c$ is used they much reach as far as finding a value for c.

A1: Accept $5x-3y+2=0$ or any integer multiple of this equation.

Question Number	Scheme	Marks
2	$\int \frac{\ln x}{x^2}\,dx = \int x^{-2}\ln x\,dx = \frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,dx$	M1A1
	$= \frac{x^{-1}}{-1}\ln x + \int x^{-2}\,dx$	
	$= \frac{x^{-1}}{-1}\ln x + \frac{x^{-1}}{-1}(+c)$	M1A1
	$\int_1^{e} \frac{\ln x}{x^2}\,dx = \left[\frac{-1}{x}\ln x - \frac{1}{x}\right]_1^{e} = \left(\frac{-1}{e}\ln e - \frac{1}{e}\right) - \left(\frac{-1}{1}\ln 1 - \frac{1}{1}\right)$	M1
	$= 1 - \frac{2}{e}$	A1
		(6)
	Alternative by substitution:	
	$u = \ln x \Rightarrow \int \frac{\ln x}{x^2}\,dx = \int \frac{u}{e^{2u}} e^{u}\,du = \int u e^{-u}\,du$	
	$\int u e^{-u}\,du = -u e^{-u} - \int -e^{-u}\,du$	M1A1
	$\int u e^{-u}\,du = -u e^{-u} - e^{-u}(+c)$	M1A1
	$\int_1^{e} \frac{\ln x}{x^2}\,dx = \left[-u e^{-u} - e^{-u}\right]_0^{1} = \left(-\frac{1}{e} - \frac{1}{e}\right) - (0-1)$	M1
	$= 1 - \frac{2}{e}$	A1

(Condone the lack of "dx" throughout)

M1: An application of integration by parts the right way around.
If the rule is quoted it must be correct. (A version appears in the formula booklet)
Must see an expression of the form $Ax^{-1}\ln x \pm B\int x^{-1}\times\frac{1}{x}\,dx$ **for this mark**

A1: A correct un-simplified (or simplified) expression e.g. $\frac{x^{-1}}{-1}\ln x - \int \frac{x^{-1}}{-1}\times\frac{1}{x}\,dx$, $\left[-\frac{1}{x}\ln x\right]_1^{e} + \int \frac{1}{x^2}\,dx$

M1: It is for 'combining' their two terms in x correctly and integrating their resulting term by adding one to the power.

A1: A completely correct integral (simplified or un-simplified)

For students who substitute in limits early, look for e.g. $\left(\frac{e^{-1}}{-1}\ln e\right) - \left(\frac{1^{-1}}{-1}\ln 1\right) + \left[\frac{x^{-1}}{-1}\right]_1^{e}$

M1: It is for substituting in the limits 1 and e (either way round) and subtracting.

A1: cso and cao for $1-\frac{2}{\mathrm{e}}$ or $-\frac{2}{\mathrm{e}}+1$. Allow e^1 for e. Accept $1+\frac{-2}{\mathrm{e}}$.

Alt 1

M1: An application of integration by parts the right way around.
If the rule is quoted it must be correct. (A version appears in the formula booklet)
Must see an expression of the form $\int u\mathrm{e}^{-u}\,\mathrm{d}u = Au\mathrm{e}^{-u} \pm \int \mathrm{e}^{-u}\,\mathrm{d}u$ **for this mark**

A1: A correct un-simplified (or simplified) expression e.g. $\int u\mathrm{e}^{-u}\,\mathrm{d}u = -u\mathrm{e}^{-u} - \int -\mathrm{e}^{-u}\,\mathrm{d}u$

M1: It is for $\int \mathrm{e}^{-u}\,\mathrm{d}u \to \mathrm{e}^{-u}$

A1: A completely correct integral (simplified or un-simplified)

M1: It is for substituting in the limits 0 and 1 (either way round) and subtracting.

A1: A1: cso and cao for $1-\frac{2}{\mathrm{e}}$ or $-\frac{2}{\mathrm{e}}+1$. Allow e^1 for e. Accept $1+\frac{-2}{\mathrm{e}}$.

Question Number	Scheme	Marks
3 (a)	$0<\mathrm{g}<3$	M1A1
		(2)
(b)	$y=\dfrac{6x}{2x+3}\Rightarrow 2xy+3y=6x\Rightarrow(6-2y)x=3y\Rightarrow x=\dfrac{3y}{(6-2y)}$	M1A1
	$\Rightarrow \mathrm{g}^{-1}(x)=\dfrac{3x}{(6-2x)}\qquad 0<x<3$	A1ft
		(3)
(c)	$\mathrm{gg}(x)=\mathrm{g}\left(\dfrac{6x}{2x+3}\right)=\dfrac{6\times\dfrac{6x}{2x+3}}{2\times\dfrac{6x}{2x+3}+3}$	M1
	$=\dfrac{6\times 6x}{2\times 6x+3(2x+3)}$	dM1
	$=\dfrac{36x}{18x+9}=\dfrac{4x}{2x+1}$	A1
		(3)
		(8 marks)

(a)

M1: For one 'end' fully correct $\mathrm{g}(x)>0$ (**not** $x>0$) or $\mathrm{g}(x)<3$ (**not** $x<3$) or both ends (incorrect) eg. accept $0\leqslant \mathrm{g}\leqslant 3$. Accept incorrect notation such as $0<x<3$ for this mark but **not** $x>0$ or $x<3$ **on their own**.

Allow use of f rather than g for the M mark but not the A mark.

A1: Accept $0<\mathrm{g}<3$, $0<y<3$, $\mathrm{g}(x)>0$ and $\mathrm{g}(x)<3$, $(0,3)$

(b)

M1: An attempt to make x or a replaced y the subject of the formula. The minimum expectation is that there is an attempt to cross multiply, expand and collect/factorise terms in x or a replaced y and obtain $x=\dfrac{\pm 3y}{(\pm 6\pm 2y)}$ or equivalent i.e. sign errors only on their algebra.

A1: $x=\dfrac{3y}{(6-2y)}$ or $\dfrac{-3y}{(2y-6)}$ or $y=\dfrac{3x}{(6-2x)}$ or $\dfrac{-3x}{(2x-6)}$ or $-\dfrac{3}{2}-\dfrac{9}{2(x-3)}$ etc. Allow $2(x-3)$ for $(2x-6)$.

A1ft: $\mathrm{g}^{-1}(x)=\dfrac{3x}{(6-2x)}\left(\text{or}\ \dfrac{-3x}{(2x-6)}\right)$ **and** $0<x<3$. You can follow through on any range from part (a) but the domain must be in terms of x not in terms of e.g. $\mathrm{g}(x)$ or $\mathrm{g}^{-1}(x)$. Do not allow $x\in\mathbb{R}$

Accept $y=\dfrac{3x}{(6-2x)}\left(\text{or}\ \dfrac{-3x}{(2x-6)}\right)\qquad 0<x<3$. Allow $2(x-3)$ for $(2x-6)$.

(c)

M1: Attempts to find $\mathrm{gg}(x)$ by finding $\mathrm{g}\left(\dfrac{6x}{2x+3}\right)$

dM1: Correct processing to obtain a single fraction of the form $\dfrac{a}{b}$. Achieved by,

- multiplying both numerator and denominator by $(2x+3)$ (must multiply both terms in the denominator)

- attempting to write the denominator as a single fraction followed by the multiplication of the numerator by an inverted denominator to obtain a single fraction of the form $\frac{a}{b}$
- attempting to write the denominator as a single fraction followed the cancellation of the same denominators

A1: $\frac{4x}{2x+1}$ cao. Ignore the presence or absence of a domain and isw once the correct answer is seen.

Question Number	Scheme	Marks
4(a)	$27(3-5x)^{-2} = 27\times\frac{1}{9}\left(1-\frac{5}{3}x\right)^{-2}$	B1, B1
	$= 3\left(1+(-2)\left(-\frac{5}{3}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5}{3}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{5}{3}x\right)^3+...\right)$	M1
	$= 3+10x+25x^2+\frac{500}{9}x^3+...$	A1, A1
		(5)
(b)	$27(3+5x)^{-2} = 27(3+5x)^{-2}$	
	$= 3-10x+25x^2-\frac{500}{9}x^3+...$	B1ft
		(1)
(c)	$27(3-x)^{-2} = 3+\frac{10}{5}x+\frac{25}{5^2}x^2+\frac{500}{9\times 5^3}x^3$	M1
	$= 3+2x+x^2+\frac{4}{9}x^3$	A1
		(2)
		(8 marks)
4(a) alt	$27(3-5x)^{-2} = 27\begin{pmatrix} 3^{-2}+(-2)\times 3^{-3}\times(-5x)+\frac{(-2)(-3)}{2}\times 3^{-4}\times(-5x)^2 \\ +\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3 \end{pmatrix}$	B1 B1 M1
	$= 27\left(\frac{1}{9}+\frac{10x}{27}+\frac{25x^2}{27}+\frac{500x^3}{243}+..\right)$	A1 A1
	$= 3+10x+25x^2+\frac{500}{9}x^3+...$	

(a)

B1: Writes down $(3-5x)^{-2}$ or uses a power of -2

B1: Takes out a factor of 3^{-2} which can be implied by $\frac{1}{9}$ or $3\times(....)$ or a first term of 3

M1: Expands $(1+kx)^{-2}$, $k \neq \pm 1$ with the structure for at least 2 terms correct (not including the "1"), from $\left(1+(-2)kx+\frac{(-2)(-3)}{2}(kx)^2+\frac{(-2)(-3)(-4)}{3!}(kx)^3+...\right)$ with or without the bracket around the kx

A1: Two of the four terms correct and simplified **but the method mark must have been awarded!**

A1: Fully correct simplified expansion $3+10x+25x^2+\frac{500}{9}x^3+...$ all on one line but isw once a correct expansion is seen.

Alternative for (a):

B1: Writes down $(3-5x)^{-2}$ or uses a power of -2

B1: For a first term of 3^{-2} in the bracket

M1: For correct structure for two of $(-2)\times3^{-3}\times(-5x)+\frac{(-2)(-3)}{2}\times3^{-4}\times(-5x)^2+\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3$

A1A1: As defined in main scheme

Allow a re-start in (b) and (c):
(b)

B1ft: The correct answer $3-10x+25x^2-\frac{500}{9}x^3+\ldots$ or if (a) was incorrect, follow through on their (a) i.e.

$A+Bx+Cx^2+Dx^3\to A-Bx+Cx^2-Dx^3$. **There must be 4 non-zero terms.** Allow follow through on an un-simplified or "un-expanded" part (a).
(c)

M1: An attempt to divide the coefficient of x by 5, the coefficient of x^2 by 5^2 and the coefficient of x^3 by 5^3, seen in at least two cases on an expansion consisting of at least 3 terms.

A1: The correct answer $3+2x+x^2+\frac{4}{9}x^3$

Or:

M1: Expands $(1+kx)^{-2}$, $k\neq\pm1$ with the structure for at least 2 terms correct (not including the "1"), from

$\left(1+(-2)kx+\frac{(-2)(-3)}{2}(kx)^2+\frac{(-2)(-3)(-4)}{3!}(kx)^3+\ldots\right)$ with or without the bracket around the kx

NB k should be $-\frac{1}{3}$

A1: The correct answer $3+2x+x^2+\frac{4}{9}x^3$

If (c) is attempted using the alternative binomial method, send to review.

Question Number	Scheme	Marks
5(a)	$\dfrac{6-5x-4x^2}{(2-x)(1+2x)} = A + \dfrac{B}{(2-x)} + \dfrac{C}{(1+2x)}$	
	$6-5x-4x^2 = A(2-x)(1+2x)+B(1+2x)+C(2-x)$	M1
	Coefficients of $x^2 \Rightarrow A=2$	B1
	Sub $x=2 \Rightarrow -20=5B \Rightarrow B=-4$, $x=-\dfrac{1}{2} \Rightarrow 7.5=2.5C \Rightarrow C=3$	dM1A1
		(4)
(b)	$\mathrm{f}(x)=2-\dfrac{4}{(2-x)}+\dfrac{3}{(1+2x)} \Rightarrow \mathrm{f}'(x)=-\dfrac{4}{(2-x)^2}-\dfrac{6}{(1+2x)^2}$	M1A1ftA1
		(3)
(c)	As $(2-x)^2>0$ and $(1+2x)^2>0 \Rightarrow \mathrm{f}'(x)<0$	B1
		(1)
		(8 marks)
5 (a) alt	$-2x^2+3x+2\overline{)-4x^2-5x+6}$ with quotient 2; $\underline{-4x^2+6x+4}$; $-11x+2$ $\dfrac{-11x+2}{(2-x)(1+2x)} = \dfrac{B}{(2-x)}+\dfrac{C}{(1+2x)}$	B1 M1
	$-11x+2=B(1+2x)+C(2-x)$ Sub $x=2 \Rightarrow -20=5B \Rightarrow B=-4$, $x=-\dfrac{1}{2} \Rightarrow 7.5=2.5C \Rightarrow C=3$	dM1 A1 **(4)**

(a)

M1: Writes $6-5x-4x^2 = A(2-x)(1+2x)+B(1+2x)+C(2-x)$ and makes an attempt to find any constant.
In the alternative it is for dividing first to obtain a quotient of ±2 and a remainder $px+q$, $p, q \neq 0$ and then writing the remainder in the form $\dfrac{B}{(2-x)}+\dfrac{C}{(1+2x)}$

B1: For the $2+....$ OR $A=2$ or quotient 2

dM1: Dependent upon previous M. It is for a correct method of finding any of the constants B or C by either substitution or correctly equating coefficients and solving simultaneous equations. In the alternative, it is for writing $px+q=B(1+2x)+C(2-x)$ and attempting to find any of the constants B or C by either substitution or equating coefficients and solving simultaneous equations

A1: For the $\dfrac{-4}{(2-x)}+\dfrac{3}{(1+2x)}$ or the values of the constants stated correctly.

(b)

M1: For an application of the chain rule. Award for $\dfrac{..}{(2-x)} \to \dfrac{..}{(2-x)^2}$ *OR* $\dfrac{..}{(1+2x)} \to \dfrac{..}{(1+2x)^2}$ where … is a constant.

A1ft: One of the two terms correct or one of the two terms correct following through from their constants i.e.

$$\frac{B}{(2-x)} \to \frac{B}{(2-x)^2} \text{ OR } \frac{C}{(1+2x)} \to \frac{-2C}{(1+2x)^2}$$

A1: Fully correct $f'(x) = -\frac{4}{(2-x)^2} - \frac{6}{(1+2x)^2}$. **Allow full marks in (b) even if *A* is incorrect in (a).**

(c)

B1: This mark depends on a fully correct derivative in (b) and a minimum of, squares are always positive, hence $f'(x) < 0$

Attempts at part (c) based on f(*x*) alone should be sent to review.

Special case: If the candidate goes back to the original function for parts (b) and (c)

(b) score for use of the quotient rule.

M1: Look for $\frac{d}{dx}\left(\frac{6-5x-4x^2}{(2-x)(1+2x)}\right) = \frac{(2-x)(1+2x)(P+Qx)-(6-5x-4x^2)(L+Mx)}{((2-x)(1+2x))^2}$

A1 A1: $$\frac{(2-x)(1+2x)(-5-8x)-(6-5x-4x^2)(3-4x)}{((2-x)(1+2x))^2}$$

(c) It is very unlikely to be correct from the quotient rule.

It would require f '(*x*) to be put in a form $f'(x) = \frac{-28\left(\left(x-\frac{4}{7}\right)^2 + \frac{54}{49}\right)}{((2-x)(1+2x))^2}$

Question Number	Scheme	Marks
6.	Not parallel as $\begin{pmatrix}6\\3\\-1\end{pmatrix}$ is not equal (or a multiple of) $\begin{pmatrix}3\\1\\2\end{pmatrix}$	B1
	Sets $\begin{pmatrix}5\\-2\\4\end{pmatrix}+\lambda\begin{pmatrix}6\\3\\-1\end{pmatrix}=\begin{pmatrix}10\\5\\-3\end{pmatrix}+\mu\begin{pmatrix}3\\1\\2\end{pmatrix}\Rightarrow \begin{array}{l}5+6\lambda=10+3\mu\\-2+3\lambda=5+1\mu\\4-1\lambda=-3+2\mu\end{array}$ Two of three	M1
	Full method to solve any two Eg (2) and (3) $\mu=2,\lambda=3$	M1, A1
	Sub into both sides of the other eqn. Eg (1) $5+6\times3$ **and** $10+3\times2$	M1
	$23\neq16$ and states that as they are not equal or do not intersect (or lines are SKEW)	A1 cso
		(6)
		(6 marks)

B1: States that lines are not parallel with a valid reason e.g.

- The direction vectors $\begin{pmatrix}6\\3\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\1\\2\end{pmatrix}$ are not equal (or multiples of each other)
- $6:3:-1\neq3:1:2$
- $\sqrt{6^2+3^2+(-1)^2}\sqrt{3^2+1^2+2^2}\neq6\times3+3\times1+2\times(-1)$ or shows $\cos\theta\neq1$ (Note $\cos\theta=0.7489$ and $\theta=41.5...^\circ$)
- $6\div3=2$ and $3\div1=3$

M1: Equates the lines. Evidence will be two of the three equations. Allow slips provided the intention to equate components is clear.
M1: Full method to solve two of the three equations to obtain values for λ *and* μ or numerical expressions for λ *and* μ
A1: Correct values of either λ *or* μ for their two equations.
Note (1) and (2) give $\lambda=\frac{16}{3}$ (*awrt* 5.3) and $\mu=9$ (*awrt* 9.0),
(1) and (3) give $\lambda=\frac{31}{15}$ (*awrt* 2.1) and $\mu=\frac{37}{15}$ (*awrt* 2.5)
M1: Substitutes both values into the third equation. Alternatively uses the value of one variable, expresses the other variable in terms of this and substitutes both into the third equation.
A1: Requires all values correct (and exact if necessary) and a statement that the lines do not intersect (or are skew). The substitution into their 3rd equation does need to be made but not fully evaluated if the results are clearly not equal. There is no need for candidates to use the word SKEW – not intersecting (or equivalent) is sufficient.
Note that a score of 011111 is possible.

Question Number	Scheme	Marks
	Examples:	
7(a)	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M0dM0A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}$ $=\frac{\cos^2 x+\sin^2 x-\cos^2 x+\sin^2 x}{2\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
		(3)
(b)	$\frac{2-2\cos 2\theta}{1+\cos 2\theta}-2=7\sec\theta$	
	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2\left(\sec^2\theta-1\right)-2=7\sec\theta$	M1
	$\Rightarrow 2\sec^2\theta-7\sec\theta-4=0$	A1
	$\Rightarrow (2\sec\theta+1)(\sec\theta-4)=0$	
	$\Rightarrow \sec\theta=-\frac{1}{2},4$	
	$\Rightarrow \cos\theta=-2,\frac{1}{4}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1,A1
		(6)
		(9 marks)
7(a) alt1	$\tan^2 x=\frac{\sin^2 x}{\cos^2 x}=\frac{\frac{1}{2}\left(1-\cos 2x\right)}{\frac{1}{2}(1+\cos 2x)}=\frac{\left(1-\cos 2x\right)}{(1+\cos 2x)}$	M1dM1A1
7(a) alt2	$\frac{1-\cos 2x}{1+\cos 2x}=\tan^2 x\Rightarrow 1-\cos 2x=\tan^2 x\left(1+\cos 2x\right)$ $1-\left(1-2\sin^2 x\right)=\tan^2 x\left(1+2\cos^2 x-1\right)$ $2\sin^2 x=\frac{\sin^2 x}{\cos^2 x}\left(2\cos^2 x\right)$ $2\sin^2 x=2\sin^2 x$	M1dM1A1

Question Number	Scheme	Marks
	Examples:	
	Which is true*	

7 (b)alt1	$2\left(\dfrac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2\dfrac{\sin^2\theta}{\cos^2\theta}-2=\dfrac{7}{\cos\theta}$ $\Rightarrow 2\sin^2\theta-2\cos^2\theta=7\cos\theta$ $\Rightarrow 2(1-\cos^2\theta)-2\cos^2\theta=7\cos\theta$	M1
	$\Rightarrow 4\cos^2\theta+7\cos\theta-2=0$	A1
	$\Rightarrow (4\cos\theta-1)(\cos\theta+2)=0$	
	$\Rightarrow \cos\theta=-2,\dfrac{1}{4}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1A1
		(6)
7 (b)alt2	$2\left(\dfrac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$2\tan^2\theta-2=7\sqrt{1+\tan^2\theta}$ $\left(2\tan^2\theta-2\right)^2=\left(7\sqrt{1+\tan^2\theta}\right)^2\Rightarrow 4\tan^4\theta-8\tan^2\theta+4=49\left(1+\tan^2\theta\right)$	M1
	$4\tan^4\theta-57\tan^2\theta-45=0$	A1
	$\left(4\tan^2\theta+3\right)\left(\tan^2\theta-15\right)=0\Rightarrow\tan^2\theta=15$	
	$\tan\theta=\sqrt{15}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1A1
		(6)

(a)

M1: Uses a correct double angle identity on the numerator or denominator **and applies this to the fraction**.

dM1: Uses correct double angle identities in the numerator and denominator leading to an expression of the form $\dfrac{a\sin^2 x}{a\cos^2 x}$

A1*: Completely correct solution. The variables must be consistent and do not accept expressions of the form '$\dfrac{\sin^2}{\cos^2}=\tan^2$' within the proof. If their working necessitates the appearance of the 2's in the numerator and denominator and they are not shown, this mark can be withheld – see examples.

(a) Alt1:

M1: Uses the identity $\tan^2 x=\dfrac{\sin^2 x}{\cos^2 x}$

dM1: Uses any two correct double angle identities.

A1*: Completely correct solution. The variables must be consistent and do not accept expressions of the form '$\frac{\sin^2}{\cos^2} = \tan^2$' within the proof.

(a) Alt 2:

M1: Multiplies both sides by the denominator of the lhs and uses any two correct double angle identities
dM1: Uses any two correct double angle identities.
A1: Obtains a correct identity and makes a conclusion.
See main scheme for some other varieties and the marks to award

(b) **Inc. Alt 1**

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$

M1: Attempts to use the trig identity $\tan^2\theta = \pm\sec^2\theta \pm 1$ to produce a quadratic equation in $\sec\theta$ or attempts to use $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ and $\sin^2\theta = \pm 1 \pm \cos^2\theta$ to produce a quadratic in $\cos\theta$.

A1: Correct 3TQ = 0. Either $2\sec^2\theta - 7\sec\theta - 4 = 0$ or $4\cos^2\theta + 7\cos\theta - 2 = 0$ or equivalent

M1: Correct method of solving 3TQ = 0 in either $\sec\theta$ or $\cos\theta$ **AND** using arccos in producing at least one answer for θ. You may need to check the roots of their quadratic if no working is seen and if the roots are incorrect and no working is shown, score M0.

A1: One of awrt $\theta = 75.5°, -75.5°$

A1: Both of awrt $\theta = 75.5°, -75.5°$

In an otherwise correct solution, deduct the final mark for extra answers in range. Ignore answers outside the range.

(b) **Alt 2**

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$

M1: Attempts to use the trig identity $\tan^2\theta = \pm\sec^2\theta \pm 1$ and squares to produce a quadratic equation in $\tan^2\theta$

A1: Correct 3TQ = 0. $4\tan^4\theta - 57\tan^2\theta - 45 = 0$ or equivalent

M1: Correct method of solving 3TQ = 0 **AND** using arctan after square root in producing at least one answer for θ. You may need to check the roots of their quadratic if no working is seen and if the roots are incorrect and no working is shown, score M0.

A1: One of awrt $\theta = 75.5°, -75.5°$

A1: Both of awrt $\theta = 75.5°, -75.5°$

For answers in radians (awrt 1.3, -1.3) deduct the final A mark.

In an otherwise correct solution, deduct the final mark for extra answers in range. Ignore answers outside the range.

Part (b) Note: If the quadratic (in sec or cos) is incorrect but fortuitously leads to the correct answers e.g. from factors of $(\sec\theta - 4)$ or $(4\cos\theta - 1)$ then the final A mark can be withheld. If the quadratic (in sec or cos) is correct but in their factorisation the $(\sec\theta - 4)$ or $(4\cos\theta - 1)$ is correct and the other factor incorrect then the final A mark can be withheld if they proceed to obtain the correct angles.
M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$ (or a method using identities (allow sign errors) to obtain an equation in terms of single angles) M1: Uses identities (allow sign errors) to produce an equation in terms of a single trig. function. A1: Correct equation M1: Solves to obtain at least one value A1: One of awrt $\theta = 75.5°, -75.5°$ A1: Both of awrt $\theta = 75.5°, -75.5°$

Question Number	Scheme	Marks
8 (a)	Strip Width =1	B1
	Area $\approx \frac{1}{2}(0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027))$ $\left(=\frac{1}{2}\times 4.7547\right)$	M1
	Awrt =2.377	A1
		(3)
(b)	$\text{Volume}=(\pi)\int_2^7 \frac{x}{x^2+1}\text{d}x=(\pi)\left[\frac{1}{2}\ln\left(x^2+1\right)\right]_2^7$	M1A1
	$=\frac{(\pi)}{2}(\ln 50-\ln 5)$	dM1
	$=\frac{\pi}{2}\ln 10$	A1
		(4)
		(7 marks)

(a)

B1: Strip width = 1 which may be implied by the $\frac{1}{2}$ in the trapezium rule

M1: For a correct attempt at using the trapezium rule.

Look for $\frac{1}{2}h((y \text{ at } x=2)+(y \text{ at } x=7)+2(\text{sum of other } y \text{ values}))$. Must be correct with no missing values and no extra values. (May be implied by a correct answer)

A1: Awrt = 2.377

Note: h = 5/6 gives Area 1.981125 and h = 5 gives 11.88675 and will probably just score the M1

Note that $\frac{1}{2}\times 1\times 0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027)$ scores B1 only unless the missing brackets are implied by a correct answer.

(b)

M1: Attempts to find $C\int \frac{x}{x^2+1}\text{d}x$ to give an expression of the form $D\left[\ln k\left(x^2+1\right)\right]$

A1: $\text{Volume}=(\pi)\int \frac{x}{x^2+1}\text{d}x=\frac{(\pi)}{2}\left[\ln\left(x^2+1\right)\right]$. Correct expression **<u>with or without</u>** π. **Ignore any limits.**

Do not allow the brackets around the $x^2 + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting x = 7 and x = 2 and subtracting either way round. Following correct work, this mark may be implied by awrt 3.62.

A1: $V=\frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi\ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V=\frac{\pi}{2}\ln|10|$

<u>By substitution 1:</u>

M1: Uses $u=x^2$ Attempts to find $C\int \frac{x}{x^2+1}\text{d}x$ to give an expression of the form $D[\ln k(u+1)]$

A1: $\text{Volume}=(\pi)\int \frac{x}{u+1}\frac{1}{2x}\text{d}u=\frac{(\pi)}{2}[\ln(u+1)]$. Correct expression **<u>with or without</u>** π. **Ignore any limits.**

Do not allow the brackets around the $u + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7^2$ and $x = 2^2$ and subtracting either way round or changing back to x and substituting x = 7 and x = 2 and subtracting either way round.

A1: $V = \frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi \ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V = \frac{\pi}{2}\ln|10|$

By substitution 2:

M1: Uses $u = x^2 + 1$ Attempts to find $C\int \frac{x}{x^2+1}\,\mathrm{d}x$ to give an expression of the form $D[\ln ku]$

A1: $\text{Volume} = (\pi)\int \frac{x}{u}\frac{1}{2x}\,\mathrm{d}u = \frac{(\pi)}{2}[\ln u]$. Correct expression **with or without** π. **Ignore any limits.**

dM1: Dependent upon previous M. It is for substituting $x = 7^2+1$ and $x = 2^2+1$ and subtracting either way round or changing back to x and substituting $x = 7$ and $x = 2$ and subtracting either way round.

A1: $V = \frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi \ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V = \frac{\pi}{2}\ln|10|$

Question Number	Scheme	Marks
9(a)	Attempts $\begin{pmatrix}2\\3\\-2\end{pmatrix}.\begin{pmatrix}5\\-6\\1\end{pmatrix}=\sqrt{\left(2^2+3^2+(-2)^2\right)}\sqrt{\left(5^2+(-6)^2+1^2\right)}\cos\left(\angle CAB\right)$	M1
	$\angle CAB=\arccos\left(-\dfrac{10}{\sqrt{17}\sqrt{62}}\right)=107.94°$	dM1A1
		(3)
(b)	Area $=\frac{1}{2}\sqrt{17}\sqrt{62}\sin\left(107.94°\right)=15.44$	M1A1
		(2)
(c)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	Uses Area $=\frac{1}{2}\lvert BC\rvert\times\lvert AD\rvert\Rightarrow 15.44=\frac{1}{2}\times\sqrt{99}\times\lvert AD\rvert\Rightarrow\lvert AD\rvert=3.10$	M1A1
		(3)
		(8 marks)
ALT (a)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	Uses cosine rule $\cos\angle CAB=\dfrac{17+62-99}{2\times\sqrt{17}\times\sqrt{62}}$	dM1
	$\Rightarrow\angle CAB=\arccos\left(-\dfrac{20}{2\sqrt{17}\sqrt{62}}\right)=107.94°$	A1
		(3)
ALT (b)	Area = $\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{\left(\frac{\sqrt{17}+\sqrt{62}+\sqrt{99}}{2}\right)\left(\frac{\sqrt{17}-\sqrt{62}+\sqrt{99}}{2}\right)\left(\frac{\sqrt{17}+\sqrt{62}-\sqrt{99}}{2}\right)\left(\frac{\sqrt{62}+\sqrt{99}-\sqrt{17}}{2}\right)}$	M1A1
Alt (b) ii	Area = $\frac{1}{2}\sqrt{\lvert AB\rvert^2\lvert AC\rvert^2-\left(\overrightarrow{AB}.\overrightarrow{AC}\right)^2}=\frac{1}{2}\sqrt{17\times62-100}=\frac{3}{2}\sqrt{106}$	M1A1
ALT (b)	**Area =** $\frac{1}{2}\lvert a\times b\rvert=\frac{1}{2}\begin{vmatrix}i&j&k\\2&3&-2\\5&-6&1\end{vmatrix}=\frac{1}{2}\lvert -9i-12j-27k\rvert=\frac{1}{2}\sqrt{9^2+12^2+27^2}=15.44$	M1A1
ALT (c)	Calculates $\lvert BC\rvert=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$	M1
	$\dfrac{\sin(\text{"}107.94\text{"})}{BC}=\dfrac{\sin B}{\text{"}\sqrt{62}\text{"}}\Rightarrow B=48.84...\Rightarrow AD=\text{"}\sqrt{17}\text{"}\sin B=3.10$	M1A1

(a)

M1: Attempts to use $\mathbf{a}.\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ where $\mathbf{a}=\pm\left(2\mathbf{i}+3\mathbf{j}-2\mathbf{k}\right)$, $\mathbf{b}=\pm\left(5\mathbf{i}-6\mathbf{j}+\mathbf{k}\right)$ I.e. correct use of Pythagoras to find and multiply the lengths together and multiplies and adds components (allow arithmetic slips) for dot product.

dM1: Dependent upon previous M1. For continuing to find $\angle CAB$ using invcos. Allow $\arccos\left(\dfrac{\pm10}{\sqrt{17}\sqrt{62}}\right)$

Implied by previous M1 + angle rounding to $108°$ or $72.1°$ (NB $\cos\theta$= -0.308…)

A1: awrt $107.94°$ only (Do **not** isw, so 107.94.. followed by 180 – 107.94… = 72.06…. scores A0

Note that the correct answer in radians is 1.8839… and scores M1dM1A0 following correct work

(b)

M1: Uses Area='$\frac{1}{2}|\mathbf{a}||\mathbf{b}|\sin C$' with $\mathbf{a}=2\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, $\mathbf{b}=5\mathbf{i}-6\mathbf{j}+\mathbf{k}$ and $C=\text{their } 107.94°$

A1: Area $=\text{awrt}\,15.44$ (Allow if $72.06°$ is used for angle *CAB*)

(c)

M1: Attempts to find the length of *BC* or BC^2 e.g. $|BC|=\sqrt{(5-2)^2+(-6-3)^2+(1--2)^2}$

Alternatively uses the cosine rule $BC^2 = \text{"}17\text{"}+\text{"}62\text{"}-2\text{"}\sqrt{17}\sqrt{62}\text{"}\cos\text{"}107.9\text{"}$

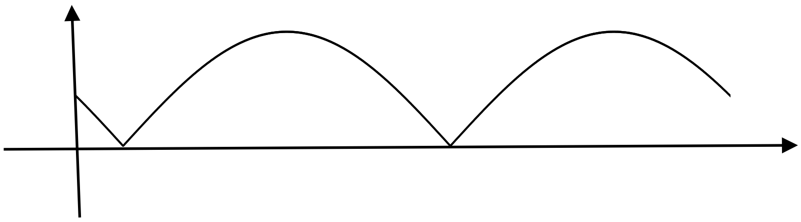
NB $BC = \sqrt{99}$ or $3\sqrt{11}$

This may be seen in (a) or (b) but must be <u>used</u> in (c) to score this mark.

M1: Attempts to use Area $=\frac{1}{2}|BC|\times|AD|$ using their area from (b) and their $|BC|$. This may be implied by their working.

A1: $|AD| = \text{awrt}\,3.10$ (Not 3.1)

Note that assuming *AD* bisects *BAC* without finding *BC* generally scores no marks.

Question Number	Scheme	Marks
10(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow(\alpha=)26.6°$	M1,A1
		(3)
(b)	 Shape	B1
	$(0,1)$	B1
	$("26.6",0)$ and $("206.6",0)$ (Allow in radians i.e. their α and π+α	B1ft
		(3)
(c)(i)	$5+'R'=5+\sqrt{5}$	B1ft
(c)(ii)	$15t-'26.6'=270\Rightarrow t=19.8$	M1,A1
		(3)
		(9 marks)

(a)

B1: $R=\sqrt{5}$

M1: For $\tan\alpha=\pm\frac{1}{2}$ or $\tan\alpha=\pm\frac{2}{1}$ or $\sin\alpha=\pm\frac{1}{"\sqrt{5}"}$ or $\cos\alpha=\pm\frac{2}{"\sqrt{5}"}$

A1: Awrt $\alpha=26.6°$

(b)

B1: Correct shape including cusps. A curve that starts downwards from the positive y-axis with two maxima. This mark is essentially for realising that the parts of the curve under the x-axis are reflected in the x-axis and for cusps that look "pointed" and not rounded.

B1: $(0,1)$ may be seen on the diagram or in the body of the script as coordinates or seen as $x = 0$, $y = 1$. If there is any ambiguity, the sketch takes precedence. Allow (1, 0) as long as it is marked in the correct place on the sketch.

B1**ft**: $(26.6,0)$ and $(206.6,0)$ or their 26.6 and 180 + their 26.6. May be seen on the sketch or in the body of the script as coordinates or seen as $y = 0$, $\theta(\text{or } x)=26.6$, $\theta(\text{or } x)=206.6$. If there is any ambiguity, the sketch takes precedence. Allow awrt 26.6 and awrt 207 or their ft values.

(c)(i)

B1ft: Follow through on $5+'R'$ including decimal answers (NB 5 + √5 = 7.24…)

(c)(ii)

M1: Attempts $15t-"26.6"=90$ or $270\Rightarrow t=...$ (Allow π/2, 3π/2 for 90, 270 if working in radians)

A1: $t=19.8$ **only**

(c)(ii) Alternative:

$f(t)=5+2\sin(15t)-\cos(15t)\Rightarrow f'(t)=30\cos(15t)+15\sin(15t)$

M1: Attempts $f'(t)=0\Rightarrow 15t=180-63.43...$ or $360-63.43$

A1: $t=19.8$ **only**

Question Number	Scheme	Marks
11.(a)	$\sqrt{\frac{3}{2}}$ or $\frac{\sqrt{3}}{\sqrt{2}}$ or $\sqrt{1.5}$ or $\frac{\sqrt{6}}{2}$	B1
		(1)
(b)	$y=(2x^2-3)\tan\left(\frac{1}{2}x\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	M1A1A1
	When $x=\alpha$ $\quad 4\alpha\tan\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}\alpha\right)=0$	
	$8\alpha\frac{\sin\left(\frac{1}{2}\alpha\right)}{\cos\left(\frac{1}{2}\alpha\right)}+(2\alpha^2-3)\times\frac{1}{\cos^2\left(\frac{1}{2}\alpha\right)}=0$	M1
	$8\alpha\sin\left(\frac{1}{2}\alpha\right)\cos\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)=0$	
	$4\alpha\sin\alpha+(2\alpha^2-3)=0$	dM1
	$2\alpha^2-3+4\alpha\sin\alpha=0$	A1*
		(6)
(c)	$x_2=\frac{3}{(2\times0.7+4\sin0.7)}$	M1
	$x_2=0.7544,\ x_3=0.7062$	A1
		(2)
(d)	Chooses interval $[0.72825, 0.72835]$	M1
	$2\times0.72825^2-3+4\times0.72825\sin0.72825=-0.0005<0$ $2\times0.72835^2-3+4\times0.72835\sin0.72835=0.00026>0$ + Reason +conclusion	A1
		(2)
		(11 marks)

(a)

B1: $x=\sqrt{\frac{3}{2}}$ or exact equivalent and no others **inside** the range. Ignore any solution outside the range so allow e.g. $x=\pm\sqrt{\frac{3}{2}}$. $\sqrt{\frac{3}{2}}$ seen unless seen in an incorrect statement e.g. $x^2=\sqrt{\frac{3}{2}}$.

(b)

M1: Attempts product rule on $y=(2x^2-3)\tan\left(\frac{1}{2}x\right)$ or $y=2x^2\tan\left(\frac{1}{2}x\right)$ if they multiply out first so look for

$\frac{\mathrm{d}(2x^2-3)}{\mathrm{d}x}\times\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{\mathrm{d}\tan\left(\frac{1}{2}x\right)}{\mathrm{d}x}$ or $\frac{\mathrm{d}\left(2x^2\right)}{\mathrm{d}x}\times\tan\left(\frac{1}{2}x\right)+2x^2\times\frac{\mathrm{d}\tan\left(\frac{1}{2}x\right)}{\mathrm{d}x}$ or e.g.

$Ax\tan\left(\frac{1}{2}x\right)+Bx^2\sec^2\frac{1}{2}x$

A1: One term correct: of $4x\tan\left(\frac{1}{2}x\right)$ or $+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ or $+(2x^2)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$

A1: $\frac{dy}{dx}=4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$. A fully correct un-simplified or simplified derivative.

M1: They need to have a $\frac{dy}{dx}$ with both $\tan\left(\frac{1}{2}x\right)$ and $\sec^2\left(\frac{1}{2}x\right)$. It is for using $\tan\left(\frac{1}{2}x\right)=\frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and

$\sec^2\left(\frac{1}{2}x\right)=\frac{1}{\cos^2\left(\frac{1}{2}x\right)}$ or $\sec^2\left(\frac{1}{2}x\right)=1+\tan^2\left(\frac{1}{2}x\right)=1+\frac{\sin^2\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ and setting $\frac{dy}{dx}=0$

This mark is for converting to an equation in sin and cos using the correct identities.

dM1: **Dependent upon both previous M's**. It is for multiplying by $\cos^2\left(\frac{1}{2}x\right)$ and using the correct identity

$2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$. This may be implied by their work but if the identity $\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)=\sin x$ is suggested, score M0.

A1*: $2\alpha^2-3+4\alpha\sin\alpha=0$ (Allow $4\alpha\sin\alpha+2\alpha^2-3=0$) . This is a printed answer so there must have been no errors, including bracketing errors. (May work in x or α or a but must be α or a for final A1)

(c)

M1: For substituting $x_1=0.7$, into the iterative formula to find x_2. It may be implied by the sight of

$x_2=\frac{3}{(2\times0.7+4\sin0.7)}$, $x_2=$ awrt 0.75 and also (if degrees were used) $x_2=$ awrt 2.1

A1: $x_2=0.7544$, $x_3=0.7062$ (awrt 4 dp)

(d)

M1: Chooses interval $[0.72825, 0.72835]$ or a smaller interval containing the root

A1: Substitutes both values into a suitable function, calculates both and follows with a reason and a conclusion.

Accept as suitable functions, $\pm(2\alpha^2-3+4\sin\alpha)$, $\pm\left(x-\frac{3}{(2x+4\sin x)}\right)$

NB: $0.72825-\frac{3}{(2\times0.72825+4\sin0.72825)}=-0.0001$, $0.72835-\frac{3}{(2\times0.72835+4\sin0.72835)}=0.00006$

Requires calculation correct to 1 sf rounded or truncated, reason (change in sign) and a minimal conclusion such as root/ turning point/ proven, hence suitable interval.

Question Number	Scheme	Marks
12 (a)	States or uses $\frac{dV}{dt}=0.4\pi-0.2\pi\sqrt{h}$	B1
	States or uses $V=4\pi h\Rightarrow\frac{dV}{dh}=4\pi$	M1A1
	Uses $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}\Rightarrow 0.4\pi-0.2\pi\sqrt{h}=4\pi\times\frac{dh}{dt}$	M1
	$20\frac{dh}{dt}=2-\sqrt{h}$	A1*
		(5)
(b)	Separates the variables $\int 20\frac{dh}{2-\sqrt{h}}=\int 1\,dt$	M1
	$(t=)\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\,dh$	A1*
		(2)
(c)	$h=(2-x)^2\Rightarrow dh=-2(2-x)dx$	B1
	$T=\int\frac{20}{2-\sqrt{h}}\,dh=\int\frac{20}{2-(2-x)}\times -2(2-x)dx$	M1
	$=\int -\frac{80}{x}+40\,dx$ **(No need for limits here)**	**d**M1A1
	$T=\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\,dh=\int_{1.6}^{0.5}-\frac{80}{x}+40\,dx=\left[-80\ln x+40x\right]_{1.6}^{0.5}$	**dd**M1
	$=\left[-80\ln 0.5+20\right]-\left[-80\ln 1.6+64\right]=80\ln 3.2-44=49\left(\text{minutes}\right)$	**ddd**M1A1
		(7)
		(14 marks)

(a)

B1: States or uses $\frac{dV}{dt}=0.4\pi-0.2\pi\sqrt{h}$. This may be embedded within the chain rule but must be identifiable as $\frac{dV}{dt}$

M1: Attempts $\frac{dV}{dh}$ from an equation for the volume of a cylinder. Accept $V=c\times\pi h\to\frac{dV}{dh}=c\pi$

A1: $\frac{dV}{dh}=4\pi$ This may be embedded within the chain rule

M1: Uses a correct form of the chain rule: Eg $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ with their $\frac{dV}{dh}$ and $\frac{dV}{dt}$

Also accept forms such as $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$

A1*: $20\frac{dh}{dt}=2-\sqrt{h}$ This is a given answer so no errors must be seen e.g. bracketing errors.

(b)

M1: Separates variables, no need for limits or integral sign e.g. $\int 20\frac{\mathrm{d}h}{2-\sqrt{h}}=\int 1\,\mathrm{d}t, \int \frac{\mathrm{d}h}{2-\sqrt{h}}=\int \frac{1}{20}\,\mathrm{d}t$

Accept for this mark an equation where the dh is in the numerator and $2-\sqrt{h}$ in the denominator on one side and dt is on the other side, with or without the integral signs. The "20" may appear on either side but must be correctly placed.

A1*: cao $(t=)\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\,\mathrm{d}h$ Correct expression as printed.

(c)

B1: Accept $h=(2-x)^2 \Rightarrow \mathrm{d}h=-2(2-x)\mathrm{d}x$ or $\frac{\mathrm{d}h}{\mathrm{d}x}=-2(2-x)$ or $\frac{\mathrm{d}h}{\mathrm{d}x}=-2\sqrt{h}$

M1: Attempt to produce an integral just in x $\int \frac{20}{2-\sqrt{h}}\,\mathrm{d}h \to \int \frac{20}{2-(2-x)}\times -2(2-x)(\mathrm{d}x)$

For this to be scored $\mathrm{d}h$ cannot just be replaced by $\mathrm{d}x$

dM1: For an integral of the form $=\int \frac{A}{x}+B\,(\mathrm{d}x)$ (This may be implied by subsequent integration)

Dependent on the previous M mark

A1: $=\int -\frac{80}{x}+40\,(\mathrm{d}x)$ (Allow $\int \frac{80}{x}-40\,(\mathrm{d}x)$ if the limits have clearly been "reversed" correctly)

(May be implied by subsequent integration – beware that this may be done by e.g. integration by parts)

ddM1: $\int \frac{A}{x}+B\,(\mathrm{d}x) \to A\ln x+Bx$. **There is no need for limits.**

Dependent on both previous M marks.

dddM1: Substitutes 0.5 and 1.6 into $A\ln x+Bx$ and subtracts either way round.

Dependent on all previous M marks.

A1: Accept $80\ln 3.2-44$ (oe) or answers rounding to 49

Question Number	Scheme	Marks
13.(a)	$t=0\Rightarrow\left(P=\right)200-\dfrac{160}{15+1}=190\Rightarrow 190\ 000$	M1A1
		(2)
(b)	$\mathrm{e}^{kt}\rightarrow a\mathrm{e}^{kt}$	M1
	$\dfrac{\mathrm{d}P}{\mathrm{d}t}=-\dfrac{(15+\mathrm{e}^{0.8t})\times 96\mathrm{e}^{0.6t}-160\mathrm{e}^{0.6t}\times 0.8\mathrm{e}^{0.8t}}{(15+\mathrm{e}^{0.8t})^2}$	M1A1
		(3)
(c)	Sets $\pm\dfrac{(15+e^{0.8t})\times 96e^{0.6t}-160e^{0.6t}\times 0.8e^{0.8t}}{(15+e^{0.8t})^2}=0\Rightarrow\mathrm{e}^{0.8t}=45$	M1A1
	$\Rightarrow T=\dfrac{\ln 45}{0.8}=4.76$	M1A1
		(4)
		(9 marks)

(a)

M1: Sets $t = 0$ in the top and bottom of the fraction, giving $\mathrm{e}^0=1$. Award if candidate attempts $200-\dfrac{160}{15+1}$ but not $\dfrac{200-160}{15+1}$ This can be awarded for a correct answer.

A1: Correct answer only. Accept 190 000 or (P =) 190 (ants).
The answer is an integer so do **not** allow awrt 190 or awrt 190 000 i.e. there should be no decimals.

(b)

M1: For showing that $\mathrm{e}^{kt}\rightarrow a\mathrm{e}^{kt}$ where a is a constant. This may be embedded within the product or quotient rule or their attempt to differentiate.

M1: For applying the quotient rule to obtain $\dfrac{\mathrm{d}P}{\mathrm{d}t}=\pm\dfrac{(15+\mathrm{e}^{0.8t})\times p\mathrm{e}^{0.6t}-q\mathrm{e}^{0.8t}\times\mathrm{e}^{0.6t}}{(15+\mathrm{e}^{0.8t})^2}$ or applying the product rule to obtain $\dfrac{\mathrm{d}P}{\mathrm{d}t}=\pm\left[A\mathrm{e}^{0.6t}(15+\mathrm{e}^{0.8t})^{-2}\times B\mathrm{e}^{0.8t}+(15+\mathrm{e}^{0.8t})^{-1}\times C\mathrm{e}^{0.6t}\right]$

Allow invisible brackets for this mark but not for the A mark below.

A1: A correct un-simplified or simplified $\dfrac{\mathrm{d}P}{\mathrm{d}t}$

Note $\dfrac{\mathrm{d}P}{\mathrm{d}t}=-\dfrac{(15+\mathrm{e}^{0.8t})\times 96\mathrm{e}^{0.6t}-160\mathrm{e}^{0.6t}\times 0.8\mathrm{e}^{0.8t}}{(15+\mathrm{e}^{0.8t})^2}$ or $-160\mathrm{e}^{0.6t}(15+\mathrm{e}^{0.8t})^{-2}\times 0.8\mathrm{e}^{0.8t}+(15+\mathrm{e}^{0.8t})^{-1}\times 128\mathrm{e}^{0.6t}$

(c) **Allow recovery here if the signs are reversed.**

M1: Sets their $\dfrac{\mathrm{d}P}{\mathrm{d}t}=0$ to obtain $p\mathrm{e}^{0.8t}=q$ or $A\mathrm{e}^{0.6t}=B\mathrm{e}^{1.4t}$ or equivalent.

A1: $\mathrm{e}^{0.8t}=45$ or $1440\mathrm{e}^{0.6t}=32\mathrm{e}^{1.4t}$ or equivalent correct equation.

M1: Having set their $\dfrac{\mathrm{d}P}{\mathrm{d}t}=0$ and obtained either $A\mathrm{e}^{\pm kt}=B$ (k may be incorrect) or $C\mathrm{e}^{\pm\alpha t}=D\mathrm{e}^{\pm\beta t}$ where $k, \alpha, \beta\neq 0$
it is awarded for the correct order of operations, taking ln's leading to $t=..$
It cannot be awarded from impossible equations Eg $\mathrm{e}^{0.8t}=-45$

A1: $T=\dfrac{\ln 45}{0.8}$ or equivalent or awrt $=4.76$

Question Number	Scheme	Marks
14 (a)	$(1,4.5)$	B1B1
		(2)
(b)	Attempts $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}=\dfrac{12\sin\theta\cos\theta}{-24\cos^2\theta\sin\theta}=\left(-\dfrac{1}{2\cos\theta}\right)$	M1A1
	Subs $\theta=\dfrac{\pi}{3}$ into $\dfrac{dy}{dx}=(-1)$	M1
	Uses gradient of normal with $(1,4.5)\Rightarrow(y-4.5)=1(x-1)$	ddM1
	$y=x+3.5$	A1*
		(5)
(c)	Attempts $\displaystyle\int y\frac{dx}{d\theta}\,d\theta=\int 6\sin^2\theta\times-24\cos^2\theta\sin\theta\,d\theta$	M1A1
	Uses $\sin^2\theta=1-\cos^2\theta\Rightarrow\displaystyle\int y\frac{dx}{d\theta}\,d\theta=\int A\left(\cos^2\theta\sin\theta-\cos^4\theta\sin\theta\right)d\theta$	dM1
	Area of trapezium= $\dfrac{1}{2}(3.5+4.5)=4$	B1
	Attempts trapezium + area under curve $=\dfrac{1}{2}(3.5+4.5)-144\displaystyle\int_{\frac{\pi}{3}}^{0}\sin^3\theta\cos^2\theta\,d\theta$	ddM1
	$Area=4+144\displaystyle\int_0^{\frac{\pi}{3}}\left(\sin\theta\cos^2\theta-\sin\theta\cos^4\theta\right)d\theta$	A1*
		(6)
(d)	Area of S $=4+144\left[-\dfrac{\cos^3\theta}{3}+\dfrac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{3}}=4+144\left(\left(-\dfrac{1}{24}+\dfrac{1}{160}\right)-\left(-\dfrac{1}{3}+\dfrac{1}{5}\right)\right)$	M1A1
	$=\dfrac{181}{10}$	A1
		(3)
		(16 marks)

(a)

B1: Either of $(1,4.5)$. Accept any exact equivalent for 4.5 e.g. 18/4, 9/2…(May be seen on the diagram)

B1: Both $(1,4.5)$. Accept any exact equivalent for 4.5 e.g. 18/4, 9/2…

(b)

M1: Attempts $\dfrac{dy}{dx}=\dfrac{dy/d\theta}{dx/d\theta}$ (Allow poor differentiation on y and/or x provided the functions are both "changed")

A1: $\dfrac{dy}{dx}=-\dfrac{12\sin\theta\cos\theta}{24\cos^2\theta\sin\theta}=\left(-\dfrac{1}{2\cos\theta}\right)$

M1: Subs $\theta=\dfrac{\pi}{3}$ into their $\dfrac{dy}{dx}=(-1)$

ddM1: **Dependent upon both previous M's**. It is for using the negative reciprocal of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with their $(1, 4.5)$ to produce an equation of a normal $\Rightarrow (y-4.5) = 1(x-1)$. Need to be careful as m = 1 is easily identifiable from the given answer.

A1*|:cso $y = x + 3.5$ (Allow $y = x + \frac{7}{2}$)

(c)

M1: Attempts to use area under a parametric curve $= \int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta = \int A\sin^3\theta\cos^2\theta(\mathrm{d}\theta)$ (Allow omission of dθ and allow un-simplified)

A1: $= \int 6\sin^2\theta \times -24\cos^2\theta\sin\theta\,\mathrm{d}\theta$

dM1: Uses the identity $\sin^2 x = 1 - \cos^2 x$ to produce an expression in an 'integrable form'

$$\int A\sin^3\theta\cos^2\theta\,\mathrm{d}\theta = \int A\sin\theta\left(1-\cos^2\theta\right)\cos^2\theta\,\mathrm{d}\theta = A\int\left(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta\right)\mathrm{d}\theta$$

Dependent on previous M mark.

B1: Area of trapezium= $\frac{1}{2}(3.5+4.5)$ or $\frac{1}{2}\times1\times8$ or $\frac{1}{2}\times8$ alternatively

$$\int_0^1 (x+3.5)\mathrm{d}x = \left[\frac{x^2}{2}+3.5x\right]_0^1 (= 0.5+3.5) = 4$$

Must see a calculation here – it is not acceptable just to state area of trapezium = 4 as this is effectively a given answer – must see a calculation.

ddM1: Attempts trapezium + area under curve. Look for $= "\frac{1}{2}(3.5+4.5)" \pm A\int_0^{\frac{\pi}{3}} \sin^3\theta\cos^2\theta\,\mathrm{d}\theta$

OR alternatively $= "\frac{1}{2}(3.5+4.5)" \pm A\int_0^{\frac{\pi}{3}} \left(\sin\theta\cos^2\theta - \sin\theta\cos^4\theta\right)\mathrm{d}\theta$

The correct limits must be seen either way around and they must be adding an attempt at the area of the trapezium.
Dependent on both previous M marks.

A1*: cso. Answer is given so **all previous marks must have been awarded** and no errors seen.

(d)

M1: $\int \sin\theta\cos^n\theta\,\mathrm{d}\theta = \pm\frac{\cos^{n+1}\theta}{n+1}$ in either term.

Or by substitution e.g. $u = \cos\theta$ to give $\pm\int\left(u^2 - u^4\right)\mathrm{d}u = \pm\frac{u^3}{3} \pm \frac{u^5}{5}$

A1: Any correct (un-simplified) answer $4 + 144\left[\left(-\frac{\cos^3\frac{\pi}{3}}{3} + \frac{\cos^5\frac{\pi}{3}}{5}\right) - \left(-\frac{\cos^3 0}{3} + \frac{\cos^5 0}{5}\right)\right]$ or appropriate limits if using substitution. If the (4 + 144) is bracketed then score A0 unless they recover.

A1: cso $= \frac{181}{10}$ (oe)

Correct answer with no working scores no marks.
Attempts at using Cartesian forms should be sent to review.

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